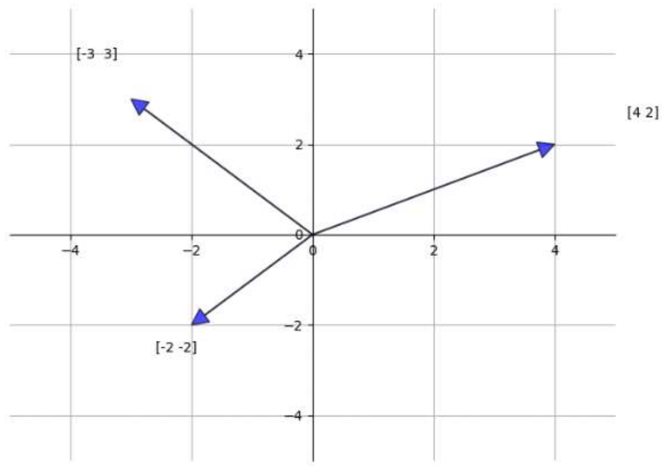
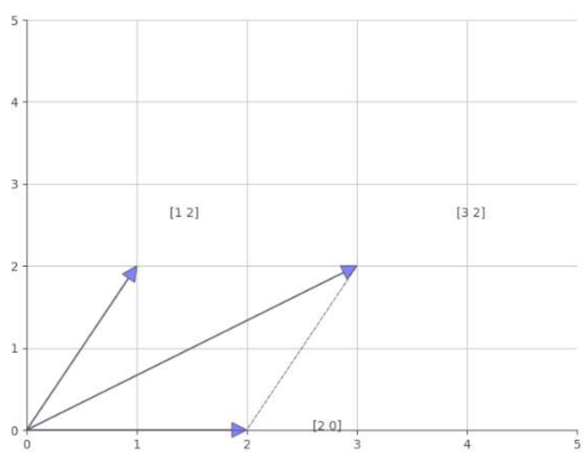
Vectors

Thursday, October 20, 2022 9:16 PM

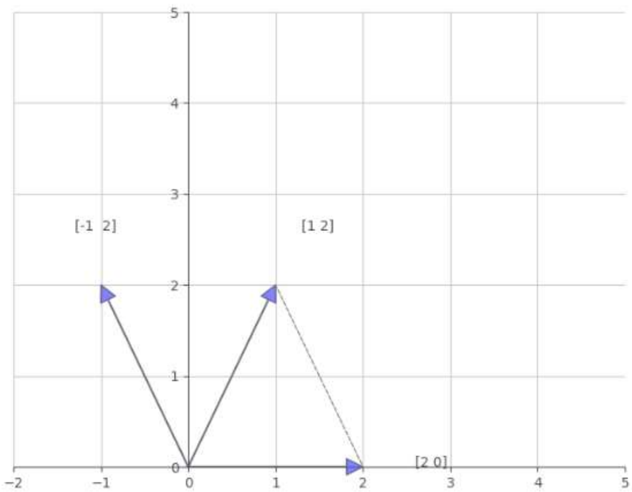
In algebra a vector is a one dimensional array of n elements.
$\vec{x} = (x_1, \dots, x_n)$
A vector in a ($\mathbb{R}^2$ $space$) would have two components:
$\mathbb{R}^2 = \{((x_1, x_2), x_1, x_2 \in \mathbb{R}\}$
$\mathbb{R}^2 = \{(x_1, x_2), x_i \in \mathbb{R}, i = 1,2\}$
$\mathbb{R}^n = \{(x_1, x_2 \cdots x_n), x_i \in \mathbb{R}, i = 1,2 \cdots n\}$
$\vec{x} = [4 \quad 2]$



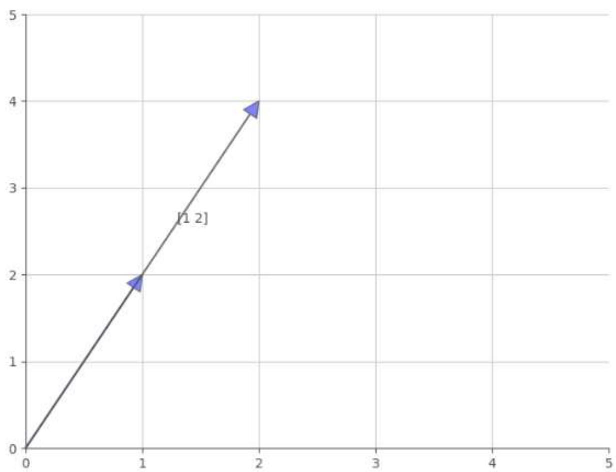
Vector Sum (Similar to matrix sum):
$\vec{x} + \vec{y} = [x_1 \quad \dots \quad x_n] + [y_1 \quad \dots \quad y_n] = [x_1 + y_1 \quad \dots \quad x_n + y_n]$



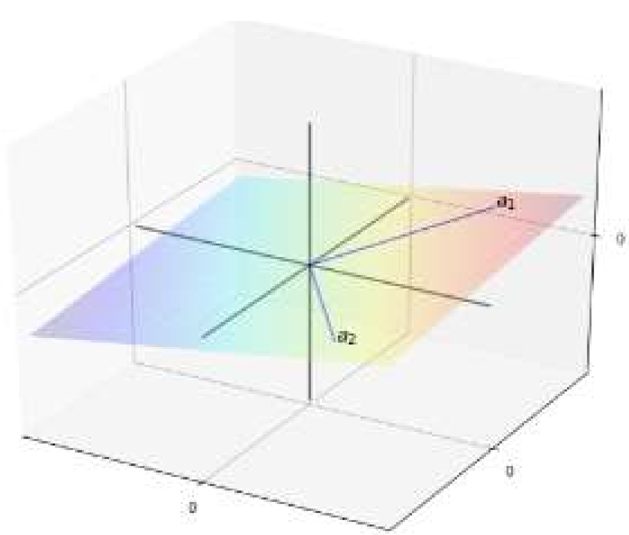
Vector Subtraction (Similar to matrix subtraction):
$\vec{x} - \vec{y} = [x_1 \quad \dots \quad x_n] + [y_1 \quad \dots \quad y_n] = [x_1 - y_1 \quad \dots \quad x_n - y_n]$

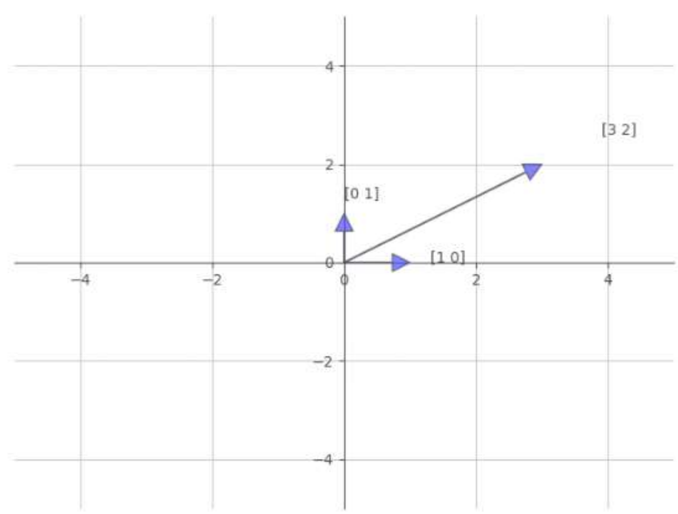


Vector Scalar Product (Similar to matrix scalar product):
$\alpha\vec{x} = [\alpha x_1 \quad \cdots \quad \alpha x_n]$



Vector Spaces:
A vector space consists in a subset of vectors, that with a linear combination of them can obtain any vector in the space. That means that a span vector depends on the linear combination of the basis vectors. So in this case all the $\mathbb{R}^2$ space will be used. But if a vector lies outside the plane, it's said that this vector will be linear indepent of the basis vectors.





Euclidean Spaces:

In Euclidean Spaces the space can be as $\mathbb{R}^n$ plane. These vectors can represent distance, length an angles

Distance: It's represented as a Metric Space, which is a set that for every point has a function that calculates the distance. That function in a $\mathbb{R}^2$ space is represented as $d(x, y) = |x - y|$, whose value always will be positive.

Length: It's represented as a Normed Space, which is a space vector that for every vector exists a scalar value that represents its length. It's notated as $||\vec{v}|| = \sqrt{\sum_{i=0}^{n} x_i}$

Vector Inner Product (Similar to matrix product):

$$\langle \vec{x} \cdot \vec{y} \rangle = \sum_{i=0}^{n} x_i y_i = X^T Y$$

If a dot product result is equal to zero, it means that both vectors are orthogonal, so with this approach we can determinate if two lines are intersecting each other. It its described in this equation:

$$\langle \vec{x} \cdot \vec{y} \rangle = ||\vec{x}|| \cdot ||\vec{y}|| \cdot \cos \alpha$$

So we can determine the vector angle with:

$$\cos \alpha = \frac{\langle \vec{x} \cdot \vec{y} \rangle}{||\vec{x}|| \cdot ||\vec{y}||}$$