Matrices

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Any vector can be seen as a matrix with only one row and multiple columns or vice verse.

Sum of matrices:
It's just for two equally shaped matrices:

$$A+B=\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}+\begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mn} \end{bmatrix}=\begin{bmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

Scalar multiplication:

$$\alpha A=\begin{bmatrix} \alpha a_{11} & \cdots & \alpha a_{1n} \\ \vdots & \ddots & \vdots \\ \alpha a_{m1} & \cdots & \alpha a_{mn} \end{bmatrix}$$

Multiplication of matrices:
It needs that m (number of columns) of the first matrix to be equal to the n (number of rows) of the second matrix.
$A(n \times m)\cdot B(m \times n)=C(n \times m)$
Example:

$$A\cdot B=\begin{bmatrix} 1 & 4 & 2 \\ 4 & 3 & 6 \end{bmatrix}_{(2\times 3)}\cdot\begin{bmatrix} 8 \\ 6 \\ 2 \end{bmatrix}_{(3\times 1)}=\begin{bmatrix} 1(8)+4(6)+2(2) \\ 4(8)+3(6)+6(2) \end{bmatrix}_{(2\times 1)}=\begin{bmatrix} 36 \\ 62 \end{bmatrix}_{(2\times 1)}$$

Transposed matrix:
It consists in exchanging the order of the matrix dimensions, if we replace a_{ij} for a_{ji} we will have a transposed matrix, that is denoted by A^T

$$A=\begin{bmatrix} 1 & 5 & 2 \\ 4 & 3 & 6 \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$A^T=\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}=\begin{bmatrix} 1 & 4 \\ 5 & 3 \\ 2 & 6 \end{bmatrix}$$

Squared matrix:
Matrix with same number of columns and rows.

$$A=\begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{44} \end{bmatrix}_{(4\times 4)}$$

Diagonal matrix:
It is represented by a matrix with values in diagonal a_{ii} and with zeros in the rest of elements. Also it must be symmetric.

$$A=\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & a_{mn} \end{bmatrix}$$

Identity matrix:
It is a diagonal matrix, whose diagonal values are one.

$$I=\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & a_{mn} \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determinant matrix:

$$\det(\mathrm{A})=|A|=\det\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\right)\therefore\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}=+a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}-a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}+a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Trace of matrix:
It is the sum of its diagonal entries:

$$tr(A)=\sum_{i=1}^{n} A_{ii}$$

Also it's the sum of its eigen values:

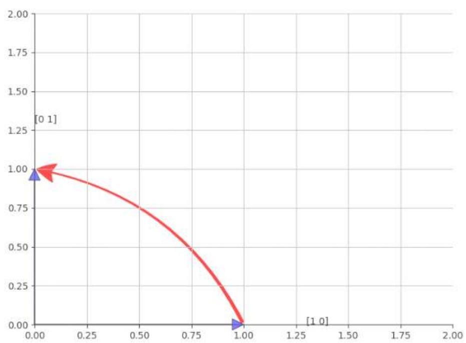
$$tr(A)=\sum_{i=1}^{n} \lambda_i$$

Inverse matrix:
It holds the property that a matrix multiplied by its inverse matrix gives the entity matrix:

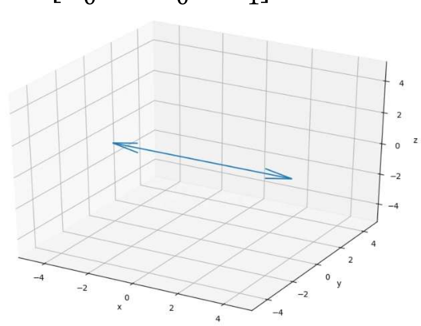
$A \cdot A^{-1} = A^{-1} \cdot A = I$

Linear maps:
A linear map is a function between vector spaces: $T: V \rightarrow W$
In a function $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$, a matrix of dimension n is transformed into a matrix of m dimension.
$$\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = A \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$$
Also it can be denoted as:
$\vec{y} = A \cdot \vec{x}$

Rotation matrix:
A matrix can make a vector rotate around the origin. It is expressed as:
$$R = \begin{bmatrix} cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$
For example: For a rotation of 90 degrees:
$$R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
Rotating vector:
$$\vec{w} = R \cdot \vec{v} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}_{(2\times 2)} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times 1}$$



In three dimensions, the three axis rotation matrices are defined as:
$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$
$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$
$$R_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

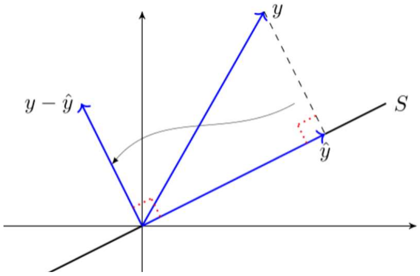


Eigen things:
A squared matrix with a non-zero vector in a vector space has eigen values and eigen vectors.
Eigen Vectors are linearly independent, also it's direction never changes, only they can be scaled.
Eigen Values scales the eigen vectors.
$A\vec{x} = \lambda\vec{x}$
Example:
$$A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$
Eigen Vector is $\vec{x}$:
$$A\vec{x} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \end{bmatrix}$$
Calculate Eigen Values and Eigen Vectors:

$\det(A - \lambda I) = 0 \rightarrow \lambda_i$
$(A - \lambda_i I)\vec{v}_{ij} = 0 \rightarrow \vec{v}_{ij}$

Pythagorean theorem:
The square value of the sum of the squared elements represents the distance of the vector.
$\|\vec{x} + \vec{y}\| = \|\vec{x}\| + \|\vec{y}\|$
$\vec{x} = (4,0)$
$\vec{y} = (0,3)$
$x_{length} = \sqrt{4^2 + 0^2 = 4}$
$y_{length} = \sqrt{0^2 + 3^2 = 3}$
$hypotenusa_{length} = 5$

Orthogonal projections:
The projection of a vector is the component of the vector in a subspace. Example given a vector $\vec{y} \in \mathbb{R}^n$ and subspace $S \subset \mathbb{R}^n$, the vector can be projected in the subspace S to give $\hat{y}$



The projection can be calculated with the basis $\vec{s}_i$ of S and the inner product between $\vec{y}$ of V $\vec{s}_i$ of S
$P_s\hat{y} = \langle\vec{y}, \vec{s}_1\rangle\vec{s}_1 + \cdots + \langle\vec{y}, \vec{s}_m\rangle\vec{s}_m$
$P\ denotes\ Projection$
$Angles\ brackes\ denote\ inner\ product$
Calculate the remaning side of the triangle:
$P_s\vec{y} = argmin\|\vec{y} - \vec{s}\|$